February 19th, 2013

Math 307

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Spring 2013

Name:. . . . .

Student ID (last 6 digits): XXX- . . . .

Midterm 1

You have 75 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 6 questions and 8 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**. Remember you also be graded on how well written your proofs are.

Good luck!

Question	Max. Points	Score
1	10	
2	10	
3	20	
4	20	
5	20	
6	20	
Total	100	

1) [10 points] Let:

$$\begin{aligned} R(x,y)&: \quad x \text{ is related to } y \\ S(x)&: \quad x \text{ is single} \\ H(x)&: \quad x \text{ is happy} \end{aligned}$$

Using the statements above, analyze the statement [which does not need to be true!]: "Every happy person is related to a person that is single".

2) [10 points] Rewrite the statement [about real numbers]:

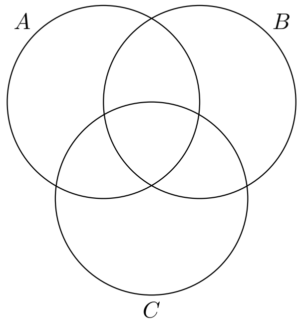
$$\neg[\exists x \text{ st } \forall y,\ [(y > x) \to (\exists z \text{ st } z^2 = y)]]$$

as a positive statement [without the "$\neg$"symbol].

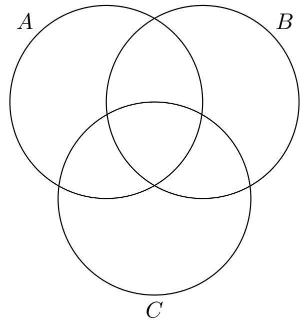
3) [20 points] Venn Diagrams:

(a) Shade the Venn Diagrams below.

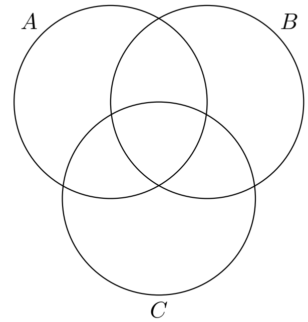
$(A \setminus B) \cup (B \cap C)$



$C \setminus A$

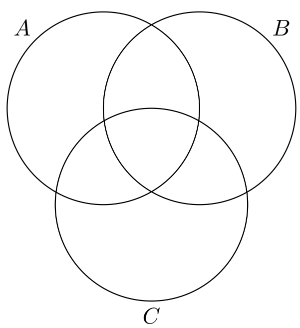


$[(A \setminus B) \cup (B \cap C)] \cap (C \setminus A)$



(b) Give concrete [and simple!] examples of sets A, B, C for which $[(A\setminus B)\cup(B\cap C)]\cap(C\setminus A)$ [the last set from (a)] is *not* the same as $(A \cap C) \cup (B \cap C)$. [**Hint:** Start by shading the Venn Diagram of the latter set.]

$(A \cap C) \cup (B \cap C)$



4) [20 points] Decide if the arguments below are valid by writing the [relevant parts of the] truth tables. [Remember that if an argument is *invalid*, then a single line of the table is enough!]

(a) If you are happy, your mother is happy. If your sister is happy, your mother is also happy. Your mother and your sister are not both happy. Therefore you are not happy.

(b)

$$\begin{array}{c} P \wedge Q \to R \\ \neg P \to \neg Q \\ Q \\ \hline \therefore R \end{array}$$

Use the table below. [You don't need to fill it all, just what is necessary.]

P	Q	R	$P \wedge Q \to R$	$\neg P \to \neg Q$	Q	R

Thus, the argument is __________________ [valid/invalid].

5) [20 points] Let A be a set and $\mathcal{F}$ be family of sets. Analyze $x \in (\bigcap \mathcal{F}) \setminus \mathscr{P}(A)$. [Your answer may only contain the symbols $\forall$, $\exists$, $\in$, $\notin$, $\vee$, $\wedge$, $\rightarrow$.]

6) [20 points] Let I be a non-empty set of indices and A_i and B_i be sets for each i in I. Show the equality below by analyzing what it means for an object x to be in each of the sets and using logical equivalences:

$$\bigcap_{i\in I}(A_i \setminus B_i) = \left(\bigcap_{i\in I} A_i\right) \setminus \left(\bigcup_{i\in I} B_i\right).$$

Scratch: