April 28th, 2009

Math 251

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Name: ....

Student ID (last 6 digits): XXX- ....

Final

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 7 questions and 12 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	10	
2	25	
3	10	
4	10	
5	10	
6	15	
7	20	
Total	100	

1) [10 points] Put the following matrix in *reduced* row echelon form:

$$\begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

2) You should be able to answer the following questions *quickly*. You do *not* need to justify your answers.

(a) [4 points] Compute $\det\left(\begin{bmatrix} 1 & 0 & 2 & -3 \\ 1 & 2 & -1 & 0 \\ 1 & 4 & -2 & 2 \\ 3 & 3 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & 0 & 3 & 1 \\ 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}\right)$.

(b) [4 points] Give the matrix that represents the reflection on the xy-plane in $\mathbb{R}^3$.

(c) [4 points] Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T(x_1, x_2, x_3) = (2x_1 - 3x_2, 0, x_2 - x_3, x_1).$$

Give $[T]$ [i.e., the matrix associated to this linear transformation].

(d) [4 points] If $W = \text{span}(\{(1, 2, -3, 1), (0, 2, 0, 2), (-1, 1, 3, 4)\})$, then the orthogonal complement of W given by what matrix space [i.e., row space, column space, or null space] of the what matrix?

(e) [4 points] Let T_A be the a linear transformation associated to the m by n matrix A. If T_A is one-to-one, then what can we say about the rank of A? [If this rank is unrelated to whether or not T_A is one-to-one, just say so.]

(f) [5 points] Let A be a 3 by 3 matrix with eigenvalues -2 and 1, with their respective eigenspaces being $\text{span}\{(1, 0, -1), (1, 1, 1)\}$ and $\text{span}\{(0, 0, 1)\}$. Give the matrix P such that $P^{-1}AP$ is diagonal, as well as $P^{-1}AP$ itself.

3) [10 points] Let

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \qquad \text{and} \qquad C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Find a matrix B such that

$$\left(A^{\mathrm{T}} + 2B\right)^{-1} = C.$$

4) [10 points] Let $\mathbf{v}$ be a vector in $\mathbb{R}^n$. Show that the set W of all vectors $\mathbf{w}$ in $\mathbb{R}^n$ such that $\mathbf{v} \cdot \mathbf{w} = 0$ is a subspace of $\mathbb{R}^n$. [**Note:** Part of this is to show that W is non-empty. To show this you just need to find a vector that you can guarantee is in W.]

5) [10 points] Let $S = \{1, x, x^2\}$ and $S' = \{1 + x, 1 + x^2, x + x^2\}$. [Both are bases of P_2.] Give the transition matrix from S to S'.

6) [15 points] Let

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Find the eigenvalues and bases for the eigenspaces of A and decide if A diagonalizable.

7) Let

$$A = \begin{bmatrix} 1 & 0 & 2 & -1 & -2 & 1 \\ -2 & 1 & -3 & 0 & 0 & -3 \\ -4 & 2 & -6 & 1 & 3 & -6 \\ -1 & 2 & 0 & -2 & -3 & -3 \end{bmatrix}.$$

Then we have:

$$A \xrightarrow{\text{row red.}} \begin{bmatrix} 1 & 0 & 2 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{and} \qquad A^{\mathrm{T}} \xrightarrow{\text{row red.}} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

You do *not* need to justify any of the items below.

(a) [5 points] Give the rank of A and the dimensions of the row space and of the column space of A?

(b) [5 points] Find a basis for the row space of A made of rows of A.

(c) [5 points] For each row of A not in the basis of the previous item, give its coordinates with respect to the basis you found.

(d) [5 points] Which vectors from the standard basis of $\mathbb{R}^6$ you can add to the vectors in the basis of the row space you found above to obtain a basis of all of $\mathbb{R}^6$?

Scratch: