October 12th, 2016

Math 307

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Fall 2016

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 3

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) [20 points] Let a, b and c be integers such that $a \mid b$. Prove that $a \mid (b + c)$ if and only if $a \mid c$.

[**Remember:** If $x, y \in \mathbb{Z}$, then $x \mid y$ denotes "x divides y", which means that there is some $z \in \mathbb{Z}$ such that $y = x \cdot z$.]

2) [20 points] Let U be any set. Prove that there is a unique $A \in \mathscr{P}(U)$ such that for all $B \in \mathscr{P}(U)$, we have $A \cap B = B$. [This was a HW problem.]

3) [20 points] Prove that if x is an integer, then $x^2 + 3x + 1$ is odd. [This one is not exactly a HW problem, but a slight twist on one.]

4) [20 points] Suppose the $I \neq \varnothing$ is a set of indices and let $\{A_i \mid i \in I\}$ and $\{B_i \mid i \in I\}$ be indexed families of sets. Prove that

$$\bigcup_{i \in I} (A_i \setminus B_i) \subseteq \left(\bigcup_{i \in I} A_i \right) \setminus \left(\bigcap_{i \in I} B_i \right).$$

[This is from the book and was done in a video. You also did something similar in HW.]

5) [20 points] Suppose $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$ are non-empty families of sets and for every $A \in \mathcal{F}$ and $B \in \mathcal{G}$, we have that $A \cup B \in \mathcal{H}$. Prove that $\bigcap \mathcal{H} \subseteq (\bigcap \mathcal{F}) \cup (\bigcap \mathcal{G})$. [This was a HW problem.]

Scratch: