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Math 251

Luís Finotti
Fall 2010

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 2

You have 60 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 9 printed pages (including this one, a page with the vector space axioms, and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	15	
2	20	
3	20	
4	20	
5	25	
Total	100	

1) [15 points] Determine if B is a basis of the corresponding vector space V or not, giving a *short* explanation. [**Hint:** None of them actually require computations!]

(a) $B = \{1 + x, 2 + 2x\}$ for $V = P_1$. [P_1 is the vector space of polynomials of degree at most 1.]

(b) $B = \{(-1, 2, 3), (0, 1, 4)\}$ for $V = \mathbb{R}^3$.

(c) $B = \{(1, 1), (-1, 1)\}$ for $V = \mathbb{R}^2$.

2) Change of basis:

(a) [12 points] Let $B = \{(1,1),(0,1)\}$ and $B' = \{(2,1),(1,1)\}$. Give the transition matrix $P_{B\to B'}$.

(b) [8 points] Let B and B' be bases of a vector space V, with transition matrix

$$P_{B\to B'} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}.$$

Then, if $[\mathbf{v}]_B = (2,-1)$, find $[\mathbf{v}]_{B'}$.

3) [20 points] Let $\mathbf{v}_1 = (0, 1, -2, 3)$, $\mathbf{v}_2 = (-2, 1, 1, 1)$, $\mathbf{v}_3 = (1, -1, 1, 0)$. To find if $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent or dependent, you need to set up a system, and depending on what happen with the system you can give the answer. Give this system *and* tell how solving it would tell you if the set is linearly independent or dependent [something like, "it is linearly dependent if the system is consistent"].

[**Note:** This is *part* of what you would have to do to find the answer, but I am saving you from actually having to solve the system! If you still don't know what I mean, just determine if the set is linearly dependent or independent.]

4) [20 points] Let

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then $B = \{A_1, A_2, A_3, A_4\}$ is a basis of $M_{2\times 2}$. [Just take my word.] Set up a linear system $A\mathbf{x} = \mathbf{b}$ whose unique solution $\mathbf{x}$ is exactly $[A]_B$ [the coordinates of A with respect to the basis B], where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

[**Note:** Again, this is *part* of what you would have to do. If you don't know what I mean, just compute $[A]_B$.]

5) Vector Spaces:

(a) [7 points] Is the set V of vectors (x, y) of $\mathbb{R}^2$ such that $x \geq 0$ a vector space [with the usual addition and scalar multiplication of vectors]? [Show work!]

(b) [8 points] The set $\mathbb{R}^2$ with the usual addition but with scalar multiplication given by $k(x, y) = (ky, kx)$ is *not* a vector space. Show this by giving a [numerical] example for which one of the axioms fails. [Use the list in the end.]

Continues on the next page!

(c) [10 points] Show that the set V of functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(1) = 0$ is a vector space [with the usual addition and scalar multiplication of functions]. [Show work!]

Vector Space Axioms

A non-empty set V with a sum and a scalar product is a vector space if it satisfies the following conditions:

0. $\mathbf{u} + \mathbf{v} \in V$ for all $\mathbf{u}, \mathbf{v} \in V$, and $k\mathbf{u} \in V$ for all $\mathbf{u} \in V$ and $k \in \mathbb{R}$;
1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ for all $\mathbf{u}, \mathbf{v} \in V$;
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ for all $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$;
3. there is $\mathbf{0} \in V$ such that $\mathbf{0} + \mathbf{u} = \mathbf{u}$ for all $\mathbf{u} \in V$;
4. given $\mathbf{u} \in V$, there exists $-\mathbf{u} \in V$ such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$;
5. $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$ for all $\mathbf{u}, \mathbf{v} \in V$ and $k \in \mathbb{R}$;
6. $(k + l)\mathbf{u} = k\mathbf{u} + l\mathbf{u}$ for all $\mathbf{u} \in V$ and $k, l \in \mathbb{R}$;
7. $k(l\mathbf{u}) = (kl)\mathbf{u}$ for all $\mathbf{u} \in V$ and $k, l \in \mathbb{R}$;
8. $1\mathbf{u} = \mathbf{u}$ for all $\mathbf{u} \in V$.

Scratch: