November 5th, 2007

Math 251

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Fall 2007

Name: ....

Student ID (last 5 digits): XXX-X...............

Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last five digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes **or calculators** are allowed on this exam!

Show all work! Correct answers without work will likely get *zero*. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	24	
2	24	
3	22	
4	30	
Total	100	

1) Answer all giving short explanations.

(a) Let V be a vector space and $\mathbf{v} \in V$. When is $\{\mathbf{v}\}$ linearly independent? [No need to explain this one.]

(b) Is the set $\{(1, 2, 3, 4, 5), (-2, -4, -6, -8, -9)\}$ linearly independent [in $\mathbb{R}^5$]?

(c) Is the set $\{(-5, \sqrt{2}), (\pi, \mathrm{e}), (\ln(3), 1/2)\}$ linearly independent [in $\mathbb{R}^2$]?

(d) Does the set $\{1 + x + x^3, -2 + x^2, 1 + x - x^2 + x^3\}$ span all of P_3 [i.e., all polynomials of degree less than or equal to 3]?

2) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator for which

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

(a) Find the matrix $[T]$ associated to the linear transformation T.

(b) Is T one-to-one? Is it onto? [Don't forget to justify!!]

3) Let $T : \mathbb{R}^m \to \mathbb{R}^n$ and W be the *range* of T. In other words, the elements of W are of the form $T(\mathbf{v})$, where $\mathbf{v} \in \mathbb{R}^m$. Show that W is a vector space.

4) Let

$$A = \begin{bmatrix} 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & -4 \\ 1 & 0 & -1 & 0 & 2 \\ 2 & 2 & 2 & 0 & 4 \end{bmatrix}.$$

(a) Find bases for the nullspace, column space, and row space of A, with the requirement that the basis for the *column* space of A is composed of columns of A. [There is no requirement for the row and null spaces.]

(b) Let S be the basis for the column space that you've found in (a). Then, for each column $\mathbf{c}_i$ of A, find $(\mathbf{c}_i)_S$ [i.e., write the coordinate vector of this column with respect to the basis S].

More space for 4).

Scratch: